

3 Properties of sea water

3.1 Physical properties (general) of sea water

3.1.0 List of symbols and abbreviations

a	surface tension, in $[\text{N m}^{-1}]$
C	electrical conductivity, as function of salinity, temperature, and pressure: $C(S, T, p)$, in $[\Omega^{-1} \text{ cm}^{-1}]$
Cl	chlorinity, in $[\text{‰}]$
Cl_v	volume chlorinity, in $[\text{kg m}^{-3}]$
c	sound velocity
c_p	specific heat at constant pressure, in $[\text{J kg}^{-1} \text{ K}^{-1}]$
c_v	specific heat at constant volume, in $[\text{J kg}^{-1} \text{ K}^{-1}]$
D	Fickian diffusion coefficient, in $[\text{m}^2 \text{ s}^{-1}]$
D_S	Soret coefficient, in $[\text{K}^{-1}]$
E_i	Chebyshev polynomial of degree i
g	gravitational acceleration
I	ionic strength
J	flux density of substance, in $[\text{kg m}^{-2} \text{ s}^{-1}]$
K	secant bulk modulus, in $[\text{dbar}]$
K_{15}	electrical conductivity ratio of sea water and standard KCl solution, $K_{15} = C(S, 15.0) / C(35, 15.0)$
L	latent heat, heat of evaporation, in $[\text{J kg}^{-1}]$
M	air concentration, in $[\text{mol m}^{-3}]$
N	Brunt-Väisälä frequency
n	concentration of substance
p	pressure, in $[\text{dbar}]$, excess over atmospheric pressure (101325 Pa)
p_r	reference pressure
p_v	vapour pressure
q	heat flux
R	electrical conductivity ratio of sea water and standard KCl solution, $R = C(S, T, p) / C(35, 15.0)$
R_p	electrical conductivity ratio, $R_p = C(S, T, p) / C(S, T, 0)$
R_T	electrical conductivity ratio, $R_T = C(S, T, 0) / C(35, T, 0)$
R_{15}	electrical conductivity ratio of sea water and Normal Water, $R_{15} = C(S, 15.0) / C(35, 15.0)$
r_T	electrical conductivity ratio, $r_T = C(35, T, 0) / C(35, 15.0)$
Δr	relative isotopic difference, eq (24)
S	salinity
T	temperature, in $[\text{°C}]$, differences also in $[\text{K}]$
T_s	thermodynamic temperature, in $[\text{K}]$
T_b	boiling point temperature
T_f	freezing point temperature
T_m	density maximum temperature
U'	shear, in $[\text{s}^{-1}]$
v	specific volume, in $[\text{m}^3 \text{ kg}^{-1}]$
z	depth, in $[\text{m}]$
α	thermal expansion coefficient, in $[\text{K}^{-1}]$
Γ	adiabatic lapse rate, in $[\text{K dbar}^{-1}]$
γ	specific heat ratio, $\gamma = c_p / c_v$
δ	specific volume anomaly, $\delta = v - v(35, 0, p)$
η	entropy
Θ	potential temperature
κ_n	adiabatic compressibility, in $[\text{dbar}^{-1}]$
κ_T	isothermal compressibility, in $[\text{dbar}^{-1}]$

λ	thermal conductivity, in [$\text{W m}^{-1} \text{K}^{-1}$]
μ	viscosity, in [N s m^{-2}]
ν	kinematic viscosity, in [$\text{m}^2 \text{s}^{-1}$]
Π	osmotic pressure, in [MPa]
ρ	density, in [kg m^{-3}]
ρ_m	maximum density
τ	tangential stress, in [Pa]
ϕ	geographic latitude
IAPSO	International Association for the Physical Sciences of the Ocean
IPTS	International Practical Temperature Scale
EOS	International Equation of State of Sea Water
PSS	Practical Salinity Scale
SMOW	Standard Mean Ocean Water

3.1.1 Thermodynamic variables

Most physical properties of sea water are similar to those of fresh water which can usually be described by functions of temperature and pressure. Since the composition of salt in water from the open ocean is constant to a good approximation (see sect. 3.4), the addition of the variable salinity is generally sufficient to obtain a complete set of 3 variables determining the physical properties of sea water. The effect of non-ideal composition is briefly discussed in subsect. 3.1.3. Most properties will differ if suspended material is contained in the sample. Only the pure sea water case, however, will be considered here. The selection of the above 3 variables is arbitrary and results from practical considerations related to measurement methods. Any of these variables could be replaced by others, like sound velocity or refractive index. Only material constants will be discussed here, thus disregarding properties that depend on the motion of the liquid, such as turbulent diffusion coefficients or turbulent viscosity.

The properties of fresh water are strongly influenced by the fact that the H_2O molecule presents an electric dipole. This results in a high dielectric constant and in the formation of water polymers that change in composition with temperature. Water, therefore, behaves abnormally compared to similar hydrogen compounds, e.g. having higher boiling and freezing temperatures, a greater specific heat, and a density maximum with respect to temperature. Properties of fresh water are summarized in various volumes of Landolt-Börnstein, „Zahlenwerte und Funktionen“. A review of the density of fresh water is given by Kell [75 K].

Differences between fresh and sea water properties, although usually being small, may have important effects, such as the density increase, the freezing temperature decrease, and the density maximum temperature decrease with growing salinity. Electrical conductivity increases enormously when adding sea salt to fresh water, and a nonvanishing osmotic pressure only occurs with salinity differences (Dietrich et al. [80 D]). Reviews of the thermodynamics of sea water can be found in Fofonoff [62 F] and Millero [74 M, 82 M, 83 M].

The presentation of physical properties in the following subsections starts with a discussion on salinity. This basic variable had to be redefined several times in the development of oceanography due to progress in measurement techniques. The chemical aspects of this topic are discussed in sect. 3.4. The second variable, the temperature, was defined by the *International Practical Temperature Scale* of 1968 (IPTS-68), on which recent formulas and tables are based (Barber [69 B]). The third variable, the pressure, refers to the excess over atmospheric pressure, being zero at the ocean surface. The presentations of sea water properties in this section only include items that are not discussed in detail later, such as acoustical (see sect. 3.2), optical (see sect. 3.3), and sea ice properties (see ch. 8).

SI units or their derivatives are used in the presentations, following the recommendations given in the SUN Report (IAPSO [79 I]). The units, decibar [dbar] or Pascal [Pa] are taken for pressure values. A pressure change of one dbar closely corresponds to a depth change of one meter in the ocean (see subsect. 3.1.3). The functional dependence on the three variables will always be written in the order salinity, temperature, and pressure, e.g. density $\rho(S, T, p)$, and symbols T and T_s will be used for Celsius and thermodynamic temperatures, respectively.

To facilitate the use of computers in calculating sea water properties from the basic variables, all functions are presented in formulas that can easily be programmed, and check values are provided to test these programmes.

The check values were computed using 35 bit words for real variables. Other word lengths will cause small deviations.

3.1.2 Salinity

Salinity is basically a measure of the mass of salt per unit mass of sea water. In oceanography salinity differences of order 10^{-5} ... 10^{-6} have to be resolved for identifying deep-sea water masses or for computing geostrophic currents from the density field. Data have to be referred to an internationally accepted standard with the above resolution to render accurate differences when comparing different sets of observations.

The classical definitions of salinity S and chlorinity Cl are given in sect. 3.4. Based on the assumption of the constancy of composition of sea salt, salinity was defined by Forch et al. [02 F] as a function of chlorinity which can be obtained from a volumetric titration (S and Cl in ‰):

$$S = 0.03 + 1.805 Cl. \quad (1)$$

Recently measurements showed that salinity may be obtained from electrical conductivity with nearly an order of magnitude better precision than from chlorinity. Salinity was therefore redefined in terms of the ratio R_{15} of sea water conductivity C to Normal Water conductivity at 15°C , and chlorinity was related to salinity by a revised formula (Wooster et al. [69 W]), henceforth called ISS-69:

$$S = -0.08996 + 28.29720 R_{15} + 12.80832 R_{15}^2 - 10.67869 R_{15}^3 + 5.98624 R_{15}^4 - 1.32311 R_{15}^5, \quad (2)$$

$$R_{15} = C(S, 15, 0) / C(35, 15, 0), \quad (3)$$

$$S = 1.80655 Cl \quad (4)$$

with S and Cl in ‰.

In order to eliminate ambiguities under conditions of ionic ratio variations, an international *Practical Salinity Scale 1978* (PSS-78) was later introduced that relates salinity by the conductivity ratio to a standard KCl solution (Lewis [80 L], UNESCO, ICES, SCOR IAPSO [81 U 1, 81 U 3]). The practical salinity S of a sample of sea water is defined in terms of the ratio K_{15} of the electrical conductivity of the sea water sample at the temperature of 15°C and the total pressure of one standard atmosphere ($101325\text{ Pa} = 10.1325\text{ dbar}$, $p=0$) to that of a KCl solution in which the mass fraction of KCl is $32.4356 \cdot 10^{-3}$, at the same temperature and pressure. The K_{15} value exactly equal to 1 corresponds, by definition, to a practical salinity S exactly equal to 35:

$$S = a_0 + a_1 K_{15}^{1/2} + a_2 K_{15} + a_3 K_{15}^{3/2} + a_4 K_{15}^2 + a_5 K_{15}^{5/2} \quad (5)$$

with

$$a_0 = 0.0080$$

$$a_1 = -0.1692$$

$$a_2 = 25.3851$$

$$a_3 = 14.0941$$

$$a_4 = -7.0261$$

$$a_5 = 2.7081$$

Range of validity:

$$2 \leq S \leq 42.$$

In situ conductivity/temperature/pressure instruments measure the value of a conductivity ratio R for ambient salinity S , temperature T , and pressure p to that of the standard KCl solution at 15°C .

$$R = \frac{C(S, T, p)}{C(35, 15, 0)} = R_p \cdot R_T \cdot r_T, \quad (6)$$

where

$$R_p = \frac{C(S, T, p)}{C(S, T, 0)}, \quad R_T = \frac{C(S, T, 0)}{C(35, T, 0)}, \quad r_T = \frac{C(35, T, 0)}{C(35, 15, 0)} \quad (7)$$

R_p , r_T , and R_T obtained in sequence constitute the procedure to reduce in situ instrument data. The terms are given by the following equations

$$R_p = 1 + \frac{p(e_1 + e_2p + e_3p^2)}{1 + d_1T + d_2T^2 + (d_3 + d_4T)R} \quad (8)$$

where

$$e_1 = 2.070 \cdot 10^{-5} \quad d_1 = 3.426 \cdot 10^{-2}$$

$$e_2 = -6.370 \cdot 10^{-10} \quad d_2 = 4.464 \cdot 10^{-4}$$

$$e_3 = 3.989 \cdot 10^{-15} \quad d_3 = 4.215 \cdot 10^{-1}$$

$$d_4 = -3.107 \cdot 10^{-3}$$

$$r_T = c_0 + c_1T + c_2T^2 + c_3T^3 + c_4T^4 \quad (9)$$

where

$$c_0 = 0.6766097$$

$$c_1 = 2.00564 \cdot 10^{-2}$$

$$c_2 = 1.104259 \cdot 10^{-4}$$

$$c_3 = -6.9698 \cdot 10^{-7}$$

$$c_4 = 1.0031 \cdot 10^{-9}$$

$$R_T = \frac{R}{R_p r_T} \quad (10)$$

$$S = a_0 + a_1R_T^{1/2} + a_2R_T + a_3R_T^{3/2} + a_4R_T^2 + a_5R_T^{5/2} + \Delta S \quad (11)$$

with coefficients $a_0 \dots a_5$ given by eq. (5), and

$$\Delta S = \frac{(T-15)}{1 + K(T-15)} (b_0 + b_1R_T^{1/2} + b_2R_T + b_3R_T^{3/2} + b_4R_T^2 + b_5R_T^{5/2}) \quad (12)$$

$$b_0 = 0.0005 \quad K = 0.0162$$

$$b_1 = -0.0056$$

$$b_2 = -0.0066$$

$$b_3 = -0.0375$$

$$b_4 = 0.0636$$

$$b_5 = -0.0144$$

Units:

S	T	p
PSS-78	IPTS-68	
	°C	dbar

Range of validity:

$$2 \leq S \leq 42$$

$$-2 \text{ °C} \leq T \leq 35 \text{ °C}$$

$$0 \leq p \leq 10\,000 \text{ dbar}$$

Only combinations of S , T , p in oceanographic range.

Check values:

R	$T [^{\circ}\text{C}]$	$p [\text{dbar}]$	R_p	r_T	R_T	S
1	15	0	1.000000	1.000000	1.000000	35.00000
1.888091	40	10 000	1.040399	1.613508	1.124739	39.99999

Estimates of accuracy:

Standard deviation of mathematical fit from measurements and maximum deviation in oceanographic range of values at low pressure equivalent to 0.0015, accuracy at high pressure estimated as 0.003.

Much of the data in the oceanic range of salinities archived before the introduction of PSS-78 are within ± 0.01 of salinity computed by the new formulas. In certain cases it may be necessary to convert earlier data to the new scale. Recommendations for data conversion procedures are given by Lewis and Perkin [81 L].

The variation of electrical conductivity at atmospheric pressure as a function of salinity and temperature is shown in Fig. 1, and its temperature and pressure dependence is represented for $S=35$ in Fig. 2. Different values of the absolute conductivity of standard sea water at 15°C have been reported, $C(35, 15, 0) = 42.914 \cdot 10^{-3} \Omega^{-1} \text{cm}^{-1}$ according to Culkin and Smith [80 C 2]. In these and the following figures the validity is restricted to the liquid phase (see subject. 3.1.8).

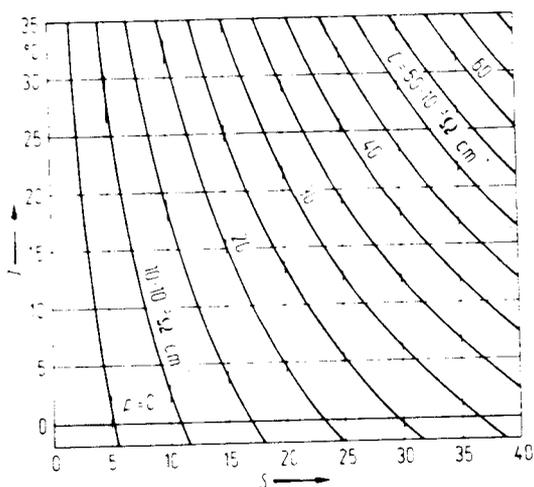


Fig. 1. Electrical conductivity C in $[10^{-3} \Omega^{-1} \text{cm}^{-1}]$ as function of practical salinity and temperature at atmospheric pressure using $C(35, 15, 0) = 42.914 \cdot 10^{-3} \Omega^{-1} \text{cm}^{-1}$. Contour interval $5 \cdot 10^{-3} \Omega^{-1} \text{cm}^{-1}$.

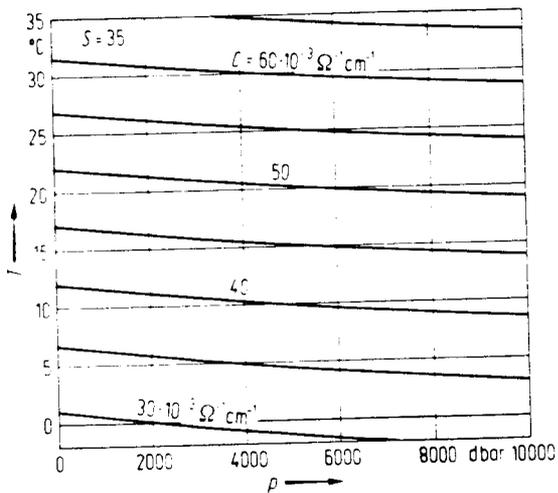


Fig. 2. Electrical conductivity C in $[10^{-3} \Omega^{-1} \text{cm}^{-1}]$ as function of pressure and temperature for $S=35$ using $C(35, 15, 0) = 42.914 \cdot 10^{-3} \Omega^{-1} \text{cm}^{-1}$. Contour interval $5 \cdot 10^{-3} \Omega^{-1} \text{cm}^{-1}$.

3.1.3 Density

Density ρ is defined as mass per unit volume. In the oceanographic literature the specific gravity or relative density (the dimensionless ratio of the density of a sea water sample to that of a reference fresh water sample) was mostly used and often called "density". This led to the dimensionless quantity $\sigma(S, T, p) = (\rho/\rho_m - 1) \cdot 10^3$, see Striggow [83 S] for a discussion. Following the SUN Report (IAPSO [79 I]) the concept of relative density should be abandoned in oceanography, and density should be expressed in $[\text{kg m}^{-3}]$. This results in the following expressions:

$$v = \frac{1}{\rho} \quad \text{specific volume of a sea water sample, in } [\text{m}^3 \text{ kg}^{-1}],$$

$$v_0 = \frac{1}{\rho_0} \quad \text{specific volume of a reference sea water, in } [\text{m}^3 \text{ kg}^{-1}], \quad v_0 \equiv v(35, 0, p),$$

$$\delta = v - v_0 \quad \text{specific volume anomaly, in } [\text{m}^3 \text{ kg}^{-1}].$$

An equivalent to the quantity σ is not defined in connection with the newly defined equation of state. The density of sea water can be related to 3 basic variables, usually salinity, temperature, and pressure. The mathematical expression for this calculation is the equation of state of sea water which has to be determined empirically. The earlier, internationally accepted equation of state has been based on direct measurements of density, chlorinity, and salinity by Forch et al. [02 F], and of sea water compressibility by Ekman [08 E]. From new measurements in different laboratories an improved equation was derived (UNESCO [81 U 2]), the *International Equation of State of Seawater*, 1980 (EOS-80). Density is given as a function of practical salinity S , temperature T , and excess pressure p . For one standard atmosphere (101325 Pa = 1.01325 bar, $p = 0$) the density $\rho(S, T, 0)$ is described by the following *One Atmosphere International Equation of State of Seawater*, 1980 (Millero and Poisson [81 M 1]).

$$\begin{aligned} \rho(S, T, 0) = \rho_w &+ (8.24493 \cdot 10^{-1} - 4.0899 \cdot 10^{-3} T \\ &+ 7.6438 \cdot 10^{-5} T^2 - 8.2467 \cdot 10^{-7} T^3 \\ &+ 5.3875 \cdot 10^{-9} T^4) S \\ &+ (-5.72466 \cdot 10^{-3} + 1.0227 \cdot 10^{-4} T \\ &- 1.6546 \cdot 10^{-6} T^2) S^{3.2} \\ &+ 4.8314 \cdot 10^{-4} S^2, \end{aligned} \quad (13)$$

where ρ_w , the density of the Standard Mean Ocean Water (SMOW) taken as pure water reference, is given by Bigg [67 B]:

$$\begin{aligned} \rho_w &= 999.842594 + 6.793952 \cdot 10^{-2} T \\ &- 9.095290 \cdot 10^{-3} T^2 + 1.001685 \cdot 10^{-4} T^3 \\ &- 1.120083 \cdot 10^{-6} T^4 + 6.536332 \cdot 10^{-9} T^5. \end{aligned} \quad (14)$$

Units:

S	T	p	$\rho \cdot \rho_w$
PSS-78	IPTS-68		
	°C	dbar	kg m^{-3}

Range of validity:

$$\begin{aligned} 0 \leq S \leq 42 \\ -2 \text{ °C} \leq T \leq 40 \text{ °C}. \end{aligned}$$

Check values:

$$\begin{aligned} S = 40, \quad T = 40 \text{ °C} \\ \rho_w = 992.2204 \text{ kg m}^{-3}, \quad \rho = 1021.6788 \text{ kg m}^{-3}. \end{aligned}$$

Estimate of accuracy relative to fresh water:

Standard deviation of mathematical fit: $3.6 \cdot 10^{-3} \text{ kg m}^{-3}$; differences between data by Millero et al. [76 M 2] and Poisson et al. [80 P]: typically no more than $\pm 5 \cdot 10^{-3} \text{ kg m}^{-3}$, maximum deviation $-7.8 \cdot 10^{-3} \text{ kg m}^{-3}$ at 30 °C and salinities between 15 and 20.

The *High Pressure International Equation of State of Seawater*, 1980 (Millero et al. [80 M]) is then given by:

$$\rho(S, T, p) = \frac{\rho(S, T, 0)}{1 - p/K(S, T, p)}, \quad (15)$$

where the secant bulk modulus K is obtained from:

$$K(S, T, p) = K(S, T, 0) + A(S, T)p + B(S, T)p^2, \quad (16)$$

$$\begin{aligned} K(S, T, 0) = & K_w + (546.746 - 6.03459 T \\ & + 1.09987 \cdot 10^{-1} T^2 \\ & - 6.1670 \cdot 10^{-4} T^3)S \\ & + (7.944 \cdot 10^{-1} + 1.6483 \cdot 10^{-1} T \\ & - 5.3009 \cdot 10^{-3} T^2)S^3, \end{aligned} \quad (17)$$

$$\begin{aligned} A(S, T) = & A_w + (2.2838 \cdot 10^{-3} - 1.0981 \cdot 10^{-5} T \\ & - 1.6078 \cdot 10^{-6} T^2)S \\ & + 1.91075 \cdot 10^{-4} S^3, \end{aligned} \quad (18)$$

$$\begin{aligned} B(S, T) = & B_w + (-9.9348 \cdot 10^{-8} + 2.0816 \cdot 10^{-9} T \\ & + 9.1697 \cdot 10^{-11} T^2)S. \end{aligned} \quad (19)$$

The pure water terms K_w , A_w , and B_w of the secant bulk modulus are given by

$$\begin{aligned} K_w = & 1.965221 \cdot 10^5 + 1484.206 T \\ & - 23.27105 T^2 + 1.360477 \cdot 10^{-1} T^3 \\ & - 5.155288 \cdot 10^{-4} T^4, \end{aligned} \quad (20)$$

$$\begin{aligned} A_w = & 3.239908 + 1.43713 \cdot 10^{-3} T \\ & + 1.16092 \cdot 10^{-4} T^2 - 5.77905 \cdot 10^{-7} T^3, \end{aligned} \quad (21)$$

$$\begin{aligned} B_w = & 8.50935 \cdot 10^{-6} - 6.12293 \cdot 10^{-7} T \\ & + 5.2787 \cdot 10^{-9} T^2. \end{aligned} \quad (22)$$

Units:

S	T	p	$\rho \cdot \rho_w$	v	K, K_w	A, A_w	B, B_w
PSS-78	IPTS-68						
	C	dbar	kg m ⁻³	m ³ kg ⁻¹	dbar		dbar ⁻¹

Check values:

$$\begin{aligned} S = 40, \quad T = 40 \text{ }^\circ\text{C}, \quad p = 10000 \text{ dbar} \\ v = 9.4355613 \cdot 10^{-4} \text{ m}^3 \text{ kg}^{-1}, \\ B_w = -7.536450 \cdot 10^{-6} \text{ dbar}^{-1}, \quad A_w = 3.446154, \\ B = -2.311202 \cdot 10^{-6} \text{ dbar}^{-1}, \quad A = 3.465376, \\ K_w = 2.260440 \cdot 10^5 \text{ dbar}, \quad K(S, T, 0) = 2.434421 \cdot 10^5 \text{ dbar}, \\ K(S, T, p) = 2.778648 \cdot 10^5 \text{ dbar}. \end{aligned}$$

Range of validity:

$$\begin{aligned} 0 \leq S \leq 42 \\ -2 \text{ }^\circ\text{C} \leq T \leq 40 \text{ }^\circ\text{C} \\ 0 \leq p \leq 10000 \text{ dbar}. \end{aligned}$$

Estimate of accuracy:

Standard deviation of mathematical fit $9 \cdot 10^{-9} \text{ m}^3 \text{ kg}^{-1}$, in oceanic range of values $5 \cdot 10^{-9} \text{ m}^3 \text{ kg}^{-1}$.
International Oceanographic Tables for the equation of state of sea water are found in UNESCO [82 U]. The salinity dependence of the density maximum temperature is discussed in subsect. 3.1.5.
Sea water density at atmospheric pressure ($p=0$) is represented in Figs. 3 and 5, and the density of sea water with $S=35$ as a function of temperature and pressure is shown in Fig. 4.

where the secant bulk modulus K is obtained from:

$$K(S, T, p) = K(S, T, 0) + A(S, T)p + B(S, T)p^2, \quad (16)$$

$$\begin{aligned} K(S, T, 0) = & K_w + (546.746 - 6.03459 T \\ & + 1.09987 \cdot 10^{-1} T^2 \\ & - 6.1670 \cdot 10^{-4} T^3)S \\ & + (7.944 \cdot 10^{-1} + 1.6483 \cdot 10^{-1} T \\ & - 5.3009 \cdot 10^{-3} T^2)S^3, \end{aligned} \quad (17)$$

$$\begin{aligned} A(S, T) = & A_w + (2.2838 \cdot 10^{-3} - 1.0981 \cdot 10^{-5} T \\ & - 1.6078 \cdot 10^{-6} T^2)S \\ & + 1.91075 \cdot 10^{-4} S^3, \end{aligned} \quad (18)$$

$$\begin{aligned} B(S, T) = & B_w + (-9.9348 \cdot 10^{-8} + 2.0816 \cdot 10^{-9} T \\ & + 9.1697 \cdot 10^{-11} T^2)S. \end{aligned} \quad (19)$$

The pure water terms K_w , A_w , and B_w of the secant bulk modulus are given by

$$\begin{aligned} K_w = & 1.965221 \cdot 10^5 + 1484.206 T \\ & - 23.27105 T^2 + 1.360477 \cdot 10^{-1} T^3 \\ & - 5.155288 \cdot 10^{-4} T^4, \end{aligned} \quad (20)$$

$$\begin{aligned} A_w = & 3.239908 + 1.43713 \cdot 10^{-3} T \\ & + 1.16092 \cdot 10^{-4} T^2 - 5.77905 \cdot 10^{-7} T^3, \end{aligned} \quad (21)$$

$$\begin{aligned} B_w = & 8.50935 \cdot 10^{-6} - 6.12293 \cdot 10^{-7} T \\ & + 5.2787 \cdot 10^{-9} T^2. \end{aligned} \quad (22)$$

Units:

S	T	p	$\rho \cdot \rho_w$	v	K, K_w	A, A_w	B, B_w
PSS-78	IPTS-68						
	C	dbar	kg m^{-3}	$\text{m}^3 \text{kg}^{-1}$	dbar		dbar^{-1}

Check values:

$$\begin{aligned} S = 40, \quad T = 40 \text{ }^\circ\text{C}, \quad p = 10000 \text{ dbar} \\ v = 9.4355613 \cdot 10^{-4} \text{ m}^3 \text{ kg}^{-1}, \\ B_w = -7.536450 \cdot 10^{-6} \text{ dbar}^{-1}, \quad A_w = 3.446154, \\ B = -2.311202 \cdot 10^{-6} \text{ dbar}^{-1}, \quad A = 3.465376, \\ K_w = 2.260440 \cdot 10^5 \text{ dbar}, \quad K(S, T, 0) = 2.434421 \cdot 10^5 \text{ dbar}, \\ K(S, T, p) = 2.778648 \cdot 10^5 \text{ dbar}. \end{aligned}$$

Range of validity:

$$\begin{aligned} 0 \leq S \leq 42 \\ -2 \text{ }^\circ\text{C} \leq T \leq 40 \text{ }^\circ\text{C} \\ 0 \leq p \leq 10000 \text{ dbar}. \end{aligned}$$

Estimate of accuracy:

Standard deviation of mathematical fit $9 \cdot 10^{-9} \text{ m}^3 \text{ kg}^{-1}$, in oceanic range of values $5 \cdot 10^{-9} \text{ m}^3 \text{ kg}^{-1}$.
International Oceanographic Tables for the equation of state of sea water are found in UNESCO [82 U]. The salinity dependence of the density maximum temperature is discussed in subsect. 3.1.5.
Sea water density at atmospheric pressure ($p=0$) is represented in Figs. 3 and 5, and the density of sea water with $S=35$ as a function of temperature and pressure is shown in Fig. 4.

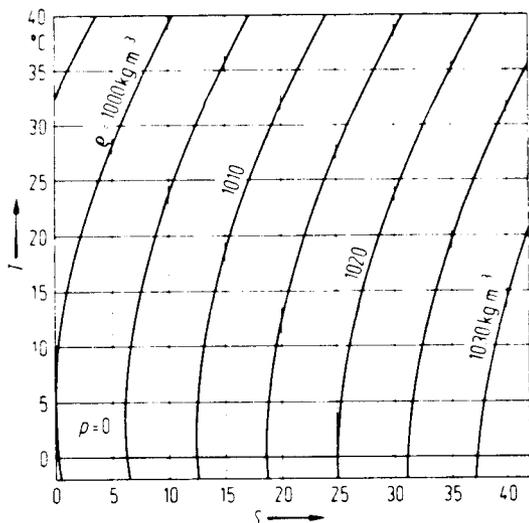


Fig. 3. Density ρ in $[\text{kg m}^{-3}]$ as function of salinity and temperature at atmospheric pressure. Contour interval 5 kg m^{-3} .

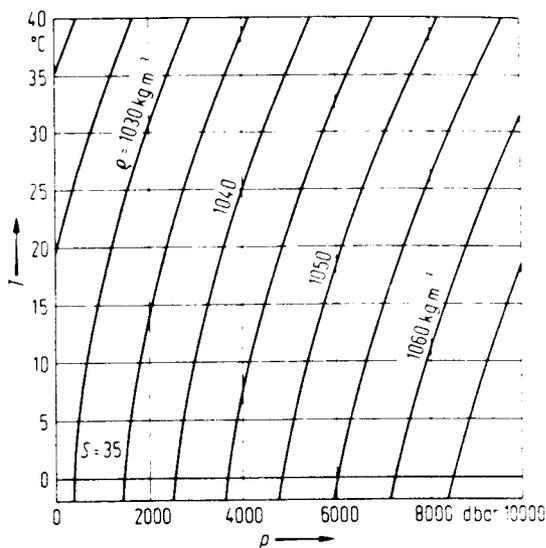


Fig. 4. Density ρ in $[\text{kg m}^{-3}]$ as function of pressure and temperature at $S = 35$. Contour interval 5 kg m^{-3} .

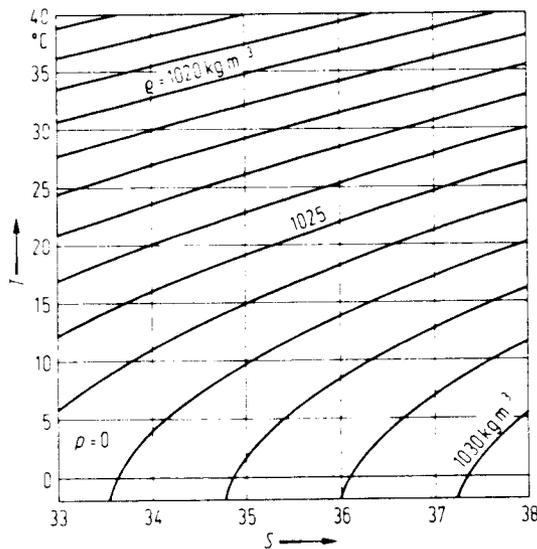


Fig. 5. Density ρ in $[\text{kg m}^{-3}]$ as function of salinity and temperature at atmospheric pressure, oceanic range. Contour interval 1 kg m^{-3} .

Even in the open ocean *variations in the composition* of salts cannot always be disregarded. For the North Pacific deep ocean a correction of 0.016 kg m^{-3} to the value calculated from the algorithm given above is necessary to obtain the true density (Millero et al. [76 M 2]). In general, such corrections are much smaller. The effect of variations in composition is larger in coastal areas. In the Baltic Sea, e.g., corrections of up to 0.045 kg m^{-3} are required (Millero and Kremling [76 M 4]; Lewis and Perkin [78 L]).

The effect of natural variations in isotopic composition on the density of water can be approximated by the following equation (Ménaché [71 M]):

$$\Delta\rho = 2.1 \cdot 10^{-4} \Delta r(^{18}\text{O}/^{16}\text{O}) + 1.5 \cdot 10^{-5} \Delta r(\text{D}/\text{H}), \quad (23)$$

where $\Delta\rho$ is the density difference at 4 °C between a water sample and Standard Mean Ocean Water (SMOW), in $[\text{kg m}^{-3}]$, and Δr the isotopic difference defined by

$$\Delta r = \left(\frac{R_{\text{sample}}}{R_{\text{standard}}} - 1 \right) \cdot 10^3, \quad (24)$$

with R representing, for sample and standard water, the respective isotopic ratios of, respectively, ^{18}O , ^{16}O and D H. In oceanic systems the density differences due to changes in the isotopic composition remain below $1 \cdot 10^{-4} \text{ kg m}^{-3}$.

Dissolved air in sea water leads to a density decrease which is small at high temperatures and when the sea water is undersaturated, and is high at saturation and at low temperatures. The density decrease $\Delta\rho$ as a function of air concentration M can be described (Millero and Emmet [76 M 1]) by:

$$\Delta\rho = -2.6 \cdot 10^{-6} M \quad (25)$$

with $\Delta\rho$ in $[\text{kg m}^{-3}]$ and M in $[\text{mol m}^{-3}]$.

For saturation, the temperature dependence of the air concentration results in a variation of $\Delta\rho$ from $3.6 \cdot 10^{-3}$ to $1.8 \cdot 10^{-3} \text{ kg m}^{-3}$ for temperatures increasing from 0 to 40 °C.

As pointed out by Fofonoff and Millard [84 F], oceanic regions of very small gradients in temperature and salinity require care in calculating the density or related properties. It is desirable to calculate the specific volume anomaly $\delta(S, T, p)$ directly and not as difference of $v(S, T, p)$ and $v(35, 0, p)$ in order to increase numerical precision. Using the notations

$$v_{35} = v(35, 0, 0) = 9.7266204 \cdot 10^{-4} \text{ m}^3 \text{ kg}^{-1},$$

$$\rho_{35} = \rho(35, 0, 0) = 1028.106331 \text{ kg m}^{-3},$$

$$\rho'(S, T) = \rho(S, T, 0) - \rho_{35},$$

$$K'(S, T, p) = K(S, T, p) - K(35, 0, p)$$

the specific volume anomaly is computed using

$$\delta(S, T, 0) = - \frac{\rho'(S, T)v_{35}}{\rho'(S, T) + \rho_{35}}, \quad (26)$$

$$K(35, 0, p) = 2.158227 \cdot 10^5 + 3.359406 p + 5.03217 \cdot 10^{-6} p^2, \quad (27)$$

$$\delta(S, T, p) = \delta(S, T, 0) \left[1 - \frac{p}{K(35, 0, p)} \right] + \frac{[v_{35} + \delta(S, T, 0)]pK'(S, T, p)}{K(35, 0, p)[K(35, 0, p) + K'(S, T, p)]} \quad (28)$$

with $K(35, 0, p)$ and p in [dbar].

$\rho'(S, T)$ is calculated from eq. (13) by changing in eq. (14) for ρ_w the coefficient 999.842594 to -28.263737 . $K'(S, T, p)$ is similarly computed from eqs. (16)–(22) by changing the following coefficients from

$$1.965221 \cdot 10^5 \text{ to } -1.93006 \cdot 10^4 \text{ in eq. (20),}$$

$$3.239908 \text{ to } -0.1194975 \text{ in eq. (21),}$$

$$8.50935 \cdot 10^{-6} \text{ to } 3.47718 \cdot 10^{-6} \text{ in eq. (22).}$$

Fofonoff and Millard [84 F] have developed an algorithm for the conversion of pressure to depth for the standard ocean ($S=35$, $T=0$ °C) which is consistent with EOS-80 [eqs. (13)ff.]. The depth z as function of geographic latitude ϕ and pressure p is given by the following formulas:

$$z = a'b, \quad (29)$$

$$a = a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4, \quad (30)$$

$$b = b_0(1 + b_1 \sin^2 \phi + b_2 \sin^4 \phi) + b_3 p, \quad (31)$$

where

$$a_1 = 9.72659 \quad b_0 = 9.780318$$

$$a_2 = -2.2512 \cdot 10^{-5} \quad b_1 = 5.2788 \cdot 10^{-3}$$

$$a_3 = 2.279 \cdot 10^{-10} \quad b_2 = 2.36 \cdot 10^{-5}$$

$$a_4 = -1.82 \cdot 10^{-15} \quad b_3 = 1.092 \cdot 10^{-6}.$$

Units:

$$\frac{p}{\text{dbar}} \quad \frac{z}{\text{m}}$$

Range of validity:

$$0 \leq p \leq 10000 \text{ dbar.}$$

Check value:

$$p = 10000 \text{ dbar, } \phi = 30^\circ: \quad z = 9712.654 \text{ m.}$$

Estimate of accuracy:

The systematic error for the standard ocean is about 0.1 m. The correction for the actual density distribution is generally less than ± 2 m in the open ocean. Fig. 6 displays the difference $|p| - |z|$ (p in [dbar] and z in [m]) calculated from eqs. (29)–(31) of the standard ocean as function of p and ϕ .

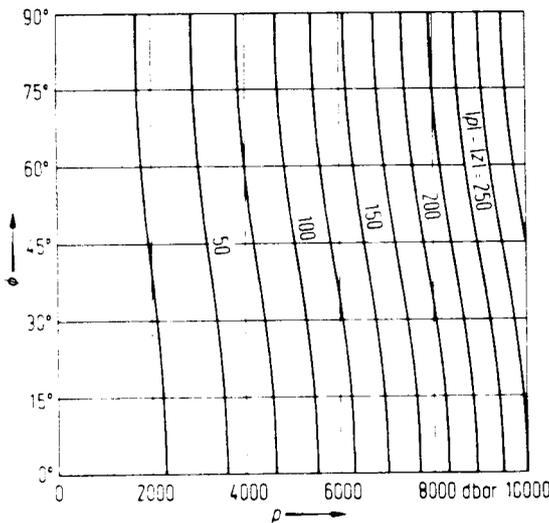


Fig. 6. Numerical difference $|p| - |z|$ of pressure p in [dbar], and depth z in [m], for the standard ocean ($S = 35$, $T = 0^\circ \text{C}$) as function of pressure and geographic latitude ϕ . z is calculated from eqs. (29)–(31). Contour interval 25.

3.1.4 Compressibility

The *isothermal* compressibility κ_T is defined by:

$$\kappa_T = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_{S,T} \tag{32}$$

With the secant bulk modulus K (the inverse “mean isothermal compressibility”) in EOS-80 [see eq. (15)] one obtains:

$$\kappa_T(S, T, p) = \frac{v(S, T, 0)}{v(S, T, p)K(S, T, p)} \left[1 - \frac{p}{K(S, T, p)} \left(\frac{\partial K(S, T, p)}{\partial p} \right)_{S,T} \right] \tag{33}$$

where

$$\left(\frac{\partial K(S, T, p)}{\partial p} \right)_{S,T} = A(S, T) + 2pB(S, T). \tag{34}$$

The isothermal compressibility can thus be calculated from eqs. (13)–(22).

Units:

S	T	p	κ_T
PSS-78	IPTS-68		
	°C	dbar	dbar^{-1}

Range of validity:

$$0 \leq S \leq 40$$

$$-2^\circ\text{C} \leq T \leq 40^\circ\text{C}$$

$$0 \leq p \leq 10000 \text{ dbar}.$$

Check value:

$$S = 40, \quad T = 40^\circ\text{C}, \quad p = 10000 \text{ dbar}$$

$$\kappa_T = 3.273850 \cdot 10^{-6} \text{ dbar}^{-1}.$$

Estimate of accuracy:

Assuming an error of $\Delta Q = 10^{-2} \text{ kg m}^{-3}$ distributed over the full pressure range yields $\Delta \kappa_T = 10^{-9} \text{ dbar}^{-1}$. This is a rough estimate. It is possible that the error increases with pressure.

With entropy η , the *adiabatic* compressibility κ_η is defined by:

$$\kappa_\eta = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_{S, \eta} \tag{35}$$

It can be obtained from the thermodynamic relation (T_s : thermodynamic temperature, α : thermal expansion coefficient at constant salinity, c_p : specific heat at constant pressure and constant salinity):

$$\kappa_\eta = \kappa_T - \frac{T_s v \alpha^2}{c_p} \tag{36}$$

With κ in $[\text{dbar}^{-1}]$ and SI units otherwise (i.e. T_s in $[\text{K}]$, α in $[\text{K}^{-1}]$, and c_p in $[\text{J kg}^{-1} \text{K}^{-1}]$):

$$\kappa_\eta = \kappa_T - 10^4 \frac{T_s v \alpha^2}{c_p} \tag{36a}$$

The formulas for $v(S, T, p)$, $\alpha(S, T, p)$, and $c_p(S, T, p)$ are given by eqs. (13)–(22), (38)–(43), and (46), (47), respectively.

Note that $\kappa_T/\kappa_\eta = c_p/c_v$; see subsect. 3.1.6.

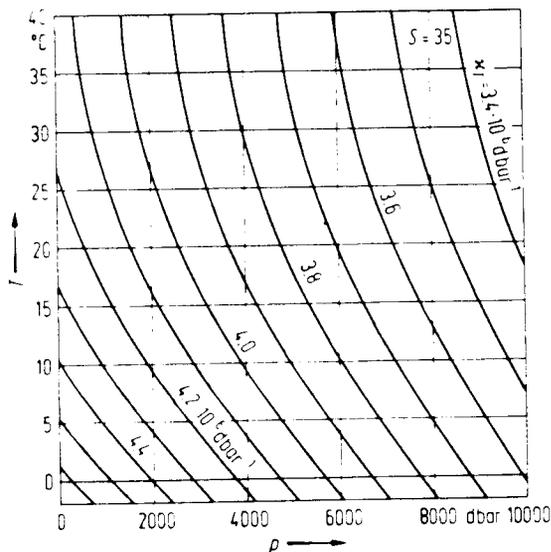
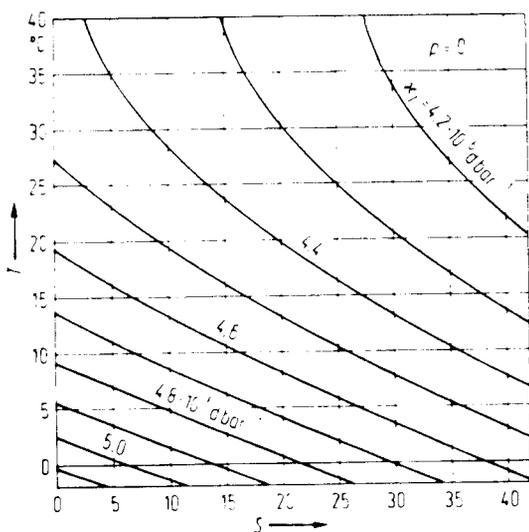


Fig. 7. Isothermal compressibility κ_T in $[10^{-6} \text{ dbar}^{-1}]$ as function of salinity and temperature at atmospheric pressure. Contour interval $10^{-7} \text{ dbar}^{-1}$.

Fig. 8. Isothermal compressibility κ_T in $[10^{-6} \text{ dbar}^{-1}]$ as function of pressure and temperature at $S = 35$. Contour interval $10^{-7} \text{ dbar}^{-1}$.

Check value:

$$S = 40, \quad T = 40^\circ\text{C}, \quad p = 10000 \text{ dbar}$$

$$\kappa_\eta = 3.139704 \cdot 10^{-6} \text{ dbar}^{-1}.$$

The isothermal compressibility for atmospheric pressure is displayed in Fig. 7, and the isothermal compressibility of sea water with $S = 35$ is shown in Fig. 8 as a function of temperature and pressure.

3.1.5 Thermal expansion, density maximum

The thermal expansion coefficient α is defined by:

$$\alpha = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_{S,p}. \quad (37)$$

Introducing the secant bulk modulus K from eq. (15) leads to:

$$\alpha(S, T, p) = pK^{-2} [1 - p/K(S, T, p)]^{-1} \left(\frac{\partial K(S, T, p)}{\partial T} \right)_{S,p}$$

$$- \rho^{-1}(S, T, 0) \left(\frac{\partial \rho(S, T, 0)}{\partial T} \right)_S. \quad (38)$$

The terms are obtained from EOS-80, i.e. from eqs. (13) and (14):

$$\left(\frac{\partial \rho(S, T, 0)}{\partial T} \right)_S = \sum_{i=0}^4 a_i T^i + S \sum_{i=0}^3 b_i T^i + S^{3/2} \sum_{i=0}^1 c_i T^i \quad (39)$$

where

$$\begin{aligned} a_0 &= 6.793952 \cdot 10^{-2} & b_0 &= -4.0899 \cdot 10^{-3} \\ a_1 &= -1.819058 \cdot 10^{-2} & b_1 &= 1.52867 \cdot 10^{-4} \\ a_2 &= 3.005055 \cdot 10^{-4} & b_2 &= -2.47401 \cdot 10^{-6} \\ a_3 &= -4.480332 \cdot 10^{-6} & b_3 &= 2.155 \cdot 10^{-8} \\ a_4 &= 3.268166 \cdot 10^{-8} & c_0 &= 1.0227 \cdot 10^{-4} \\ & & c_1 &= -3.3092 \cdot 10^{-6} \end{aligned}$$

and, according to eq. (16),

$$\left(\frac{\partial K(S, T, p)}{\partial T} \right)_{S,p} = \left(\frac{\partial K(S, T, 0)}{\partial T} \right)_S + \left(\frac{\partial A(S, T)}{\partial T} \right)_S p + \left(\frac{\partial B(S, T)}{\partial T} \right)_S p^2, \quad (40)$$

while eqs. (17)–(22) yield

$$\left(\frac{\partial K(S, T, 0)}{\partial T} \right)_S = \sum_{i=0}^3 d_i T^i + S \sum_{i=0}^2 e_i T^i + S^{3/2} \sum_{i=0}^1 f_i T^i, \quad (41)$$

where

$$\begin{aligned} d_0 &= 1.484206 \cdot 10^3 & e_0 &= -6.03459 \\ d_1 &= -4.65421 \cdot 10^1 & e_1 &= 2.19974 \cdot 10^{-1} \\ d_2 &= 4.081431 \cdot 10^{-1} & e_2 &= -1.8501 \cdot 10^{-3} \\ d_3 &= -2.0621152 \cdot 10^{-3} & f_0 &= 1.6483 \cdot 10^{-1} \\ & & f_1 &= -1.06018 \cdot 10^{-2} \end{aligned}$$

$$\left(\frac{\partial A(S, T)}{\partial T} \right)_S = \sum_{i=0}^2 g_i T^i + S \sum_{i=0}^1 h_i T^i, \quad (42)$$

where

$$\begin{aligned}
 g_0 &= 1.43713 \cdot 10^{-3} & h_0 &= -1.0981 \cdot 10^{-5} \\
 g_1 &= 2.32184 \cdot 10^{-4} & h_1 &= -3.2156 \cdot 10^{-6} \\
 g_2 &= -1.733715 \cdot 10^{-6}
 \end{aligned}$$

$$\left(\frac{\partial B(S, T)}{\partial T} \right)_S = \sum_{i=0}^2 k_i T^i + S \sum_{i=0}^2 l_i T^i \tag{43}$$

where

$$\begin{aligned}
 k_0 &= -6.12293 \cdot 10^{-7} & l_0 &= 2.0816 \cdot 10^{-9} \\
 k_1 &= 1.05574 \cdot 10^{-8} & l_1 &= 1.83394 \cdot 10^{-10}
 \end{aligned}$$

Units:

S	T	p	α
PSS-78	IPTS-68		
	°C	dbar	K ⁻¹

Range of validity:

$$\begin{aligned}
 0 &\leq S \leq 40 \\
 -2 \text{ °C} &\leq T \leq 40 \text{ °C} \\
 0 &\leq p \leq 10000 \text{ dbar}
 \end{aligned}$$

Check value:

$$\begin{aligned}
 S &= 40, \quad T = 40 \text{ °C}, \quad p = 10000 \text{ dbar} \\
 \alpha &= 4.1810597 \cdot 10^{-4} \text{ K}^{-1}
 \end{aligned}$$

Estimate of accuracy:

Assuming an error of $\Delta\rho = 5 \cdot 10^{-3} \text{ kg m}^{-3}$ occurring over an interval of 10 °C (see subsect. 3.1.3) yields $\Delta\alpha = 5 \cdot 10^{-7} \text{ K}^{-1}$.

This is a rough estimate. Also see below.

The thermal expansion coefficient of sea water for atmospheric pressure is represented in Fig. 9, the temperature and pressure dependence of $S = 35$ sea water is given in Fig. 10.

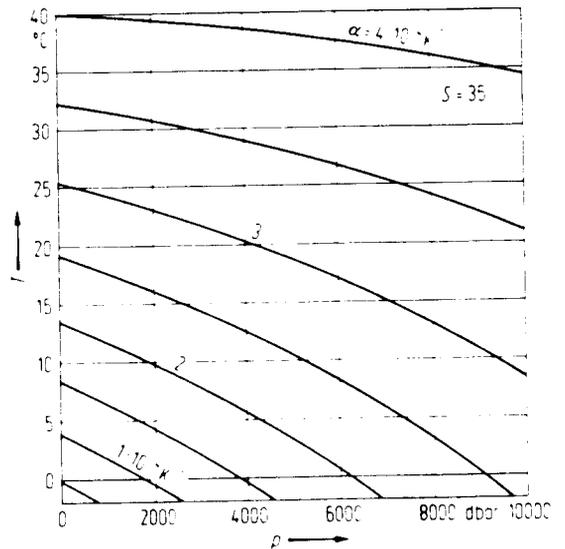
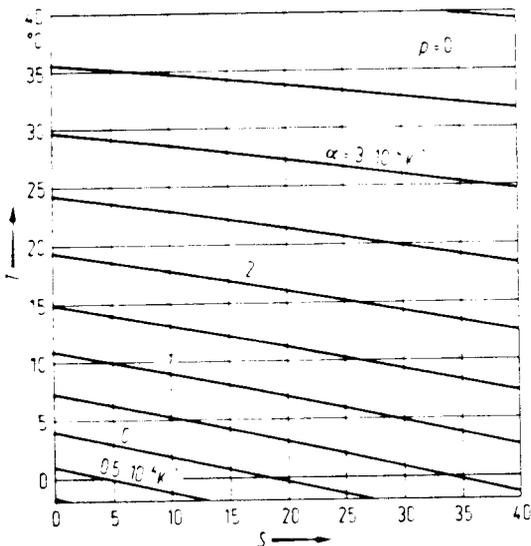


Fig. 9. Thermal expansion coefficient α in $[10^{-4} \text{ K}^{-1}]$ as function of salinity and temperature at atmospheric pressure. Contour interval $5 \cdot 10^{-5} \text{ K}^{-1}$.

Fig. 10. Thermal expansion coefficient α in $[10^{-4} \text{ K}^{-1}]$ as function of pressure and temperature for $S = 35$. Contour interval $5 \cdot 10^{-5} \text{ K}^{-1}$.

For a number of reasons the above equation of state cannot be expected to yield accurate values of the density maximum temperature. The principal problem is the slow change of density and specific volume with temperature near ρ_m . It is therefore suggested to follow Caldwell [78 C], who used the fact that, because of the thermal expansion coefficient α becoming zero, the adiabatic lapse rate Γ (see subject. 3.1.7) also becomes zero at the density maximum temperature,

$$\Gamma = \frac{T_1 v}{c_p} \alpha. \quad (44)$$

A measurement of Γ versus pressure for given S and T leads to the pressure at which Γ passes through zero and thus to the density maximum temperature T_m for a certain S and the respective p . The following formula is derived for the density maximum temperature:

$$T_m = T_0 - aS - bp(1 + cS)(1 + dp), \quad (45)$$

where

$$T_0 = 3.982$$

$$a = 0.2229$$

$$b = 2.004 \cdot 10^{-2}$$

$$c = 3.76 \cdot 10^{-3}$$

$$d = 4.02 \cdot 10^{-5}.$$

Units:

S	p	T_m
ISS-69		
	dbar	°C

Range of validity:

$$0 \leq S \leq 30$$

$$0 \leq p \leq 4000 \text{ dbar}.$$

Check value:

$$S = 25, \quad p = 1500 \text{ dbar}$$

$$T_m = -5.0773644 \text{ } ^\circ\text{C}.$$

Estimate of accuracy:

Standard deviation of data from formula 0.044 C.

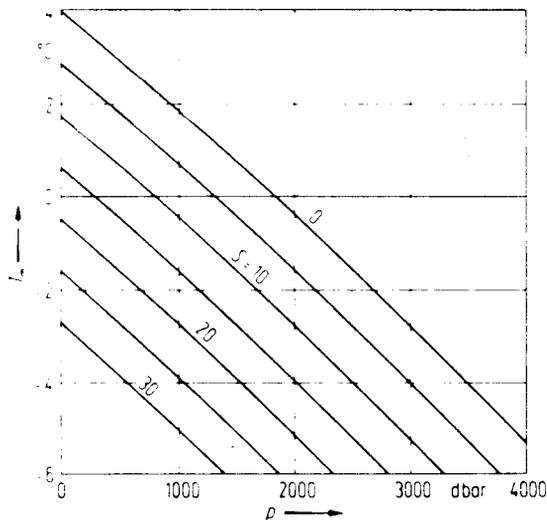


Fig. 11. Density maximum temperature T_m as function of pressure for various salinities between $S = 0$ and $S = 30$.

The values of T_m as derived from Caldwell's formula coincide with those from EOS-80 at $S=0, p=0$ and are lower by $0.133\text{ }^\circ\text{C}$ at $S=30, p=0$. At elevated pressure the differences are even larger. The error introduced by the different salinity scales is only of the order $0.01\text{ }^\circ\text{C}$. The corresponding density maximum temperature T_m is shown in Fig. 11 as a function of pressure p for various salinity values.

3.1.6 Specific heat

The specific heat is the amount of heat required to raise the temperature of a unit mass by one unit of temperature. The values given by Millero et al. [73 M] are based on measurements of the specific heat at constant pressure c_p as a function of chlorinity and temperature. Converting with $S=1.80655\text{ Cl}$ (eq. (4)) Millero et al. [81 M 2] arrived at the following formulas for specific heat at atmospheric pressure:

$$c_p(S, T, 0) = \sum_{i=0}^4 a_i T^i + S \sum_{i=0}^2 b_i T^i + S^2 \sum_{i=0}^2 c_i T^i, \quad (46)$$

where

$a_0 = 4.2174 \cdot 10^3$	$b_0 = -7.643575$	$c_0 = 0.1770383$
$a_1 = -3.720283$	$b_1 = 0.1072763$	$c_1 = -4.07718 \cdot 10^{-3}$
$a_2 = 0.1412855$	$b_2 = -1.38385 \cdot 10^{-3}$	$c_2 = 5.148 \cdot 10^{-5}$
$a_3 = -2.654387 \cdot 10^{-3}$		
$a_4 = 2.093236 \cdot 10^{-5}$		

Units:

S	T	c_p
ISS-69	IPTS-68	
$^\circ\text{C}$		$\text{J kg}^{-1} \text{K}^{-1}$

Range of validity:

$$0 \leq S \leq 40$$

$$0 \leq T \leq 35\text{ }^\circ\text{C}.$$

Check value:

$$S = 40, \quad T = 40\text{ }^\circ\text{C}$$

$$c_p = 3980.0513\text{ J kg}^{-1} \text{K}^{-1}.$$

Estimate of accuracy:

Standard deviation of data from equation $0.5\text{ J kg}^{-1} \text{K}^{-1}$; typical deviation from former investigations $2\text{ J kg}^{-1} \text{K}^{-1}$.

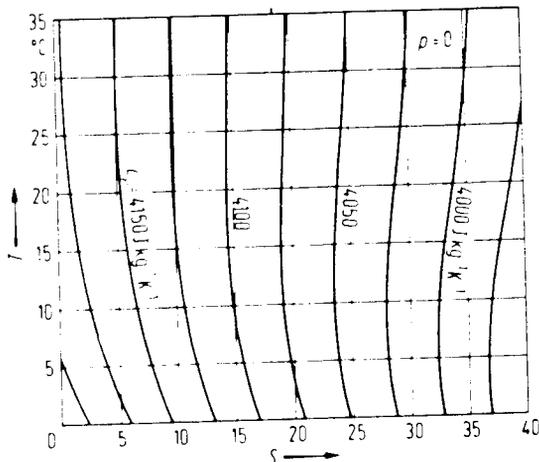


Fig. 12. Specific heat at constant pressure c_p in $[\text{J kg}^{-1} \text{K}^{-1}]$ as function of salinity and temperature at atmospheric pressure. Contour interval $25\text{ J kg}^{-1} \text{K}^{-1}$.

The dependence of c_p at atmospheric pressure on salinity and temperature is given in Fig. 12.

The pressure dependence of c_p has not been measured directly, but can be obtained from the integration of

$$\left(\frac{\partial c_p}{\partial p}\right)_{S,T} = -T \left(\frac{\partial^2 v}{\partial T^2}\right)_{S,p}$$

With $v(S, T, p)$ from EOS-80 (eqs. (13)–(22)), Fofonoff and Millard [84F] obtained:

$$\begin{aligned} c_p(S, T, p) = & c_p(S, T, 0) + p \sum_{i=0}^4 a_i T^i \\ & + p^2 \sum_{i=0}^4 b_i T^i + p^3 \sum_{i=0}^3 c_i T^i \\ & + Sp \sum_{i=0}^4 d_i T^i + S^{3/2} p \sum_{i=0}^2 e_i T^i \\ & + Sp^2 \sum_{i=0}^3 f_i T^i + g_0 S^{3/2} p^2 \\ & + Sp^3 \sum_{i=0}^2 h_i T^i + k_1 TS^3 p^3. \end{aligned} \quad (47)$$

$a_0 = -4.9592 \cdot 10^{-2}$	$b_0 = 2.4931 \cdot 10^{-6}$
$a_1 = 1.45747 \cdot 10^{-3}$	$b_1 = -1.08645 \cdot 10^{-7}$
$a_2 = -3.13885 \cdot 10^{-5}$	$b_2 = 2.87533 \cdot 10^{-9}$
$a_3 = 2.0357 \cdot 10^{-7}$	$b_3 = -4.0027 \cdot 10^{-11}$
$a_4 = 1.7168 \cdot 10^{-9}$	$b_4 = 2.2956 \cdot 10^{-13}$
$c_0 = -5.422 \cdot 10^{-11}$	$d_0 = 4.9247 \cdot 10^{-4}$
$c_1 = 2.6380 \cdot 10^{-12}$	$d_1 = -1.28315 \cdot 10^{-5}$
$c_2 = -6.5637 \cdot 10^{-14}$	$d_2 = 9.802 \cdot 10^{-8}$
$c_3 = 6.136 \cdot 10^{-16}$	$d_3 = 2.5941 \cdot 10^{-9}$
$e_0 = -1.2331 \cdot 10^{-5}$	$d_4 = -2.9179 \cdot 10^{-11}$
$e_1 = -1.517 \cdot 10^{-7}$	$f_0 = -2.9558 \cdot 10^{-8}$
$e_2 = 3.122 \cdot 10^{-9}$	$f_1 = 1.17054 \cdot 10^{-9}$
$g_0 = 9.971 \cdot 10^{-10}$	$f_2 = -2.3905 \cdot 10^{-11}$
$h_0 = 5.540 \cdot 10^{-13}$	$f_3 = 1.8448 \cdot 10^{-13}$
$h_1 = -1.7682 \cdot 10^{-14}$	$k_1 = -1.4300 \cdot 10^{-15}$
$h_2 = 3.513 \cdot 10^{-16}$	

S ISS-69	T IPTS-68	p	c_p
C		dbar	$\text{J kg}^{-1} \text{K}^{-1}$

Range of validity:

$$0 \leq S \leq 40$$

$$0 \leq T \leq 35 \text{ C}$$

$$0 \leq p \leq 10000 \text{ dbar}$$

Check value:

$$S = 40, \quad T = 40 \text{ C}, \quad p = 10000 \text{ dbar}$$

$$c_p(S, T, p) = 3940.4995 \text{ J kg}^{-1} \text{K}^{-1}$$

38495

Estimate of accuracy:

The standard deviation of the polynomial from the direct integration is less than $0.1 \text{ J kg}^{-1} \text{ K}^{-1}$. An estimate of systematic errors is not available in the original literature.

$c_p(35, T, p)$ is represented graphically in Fig. 13.

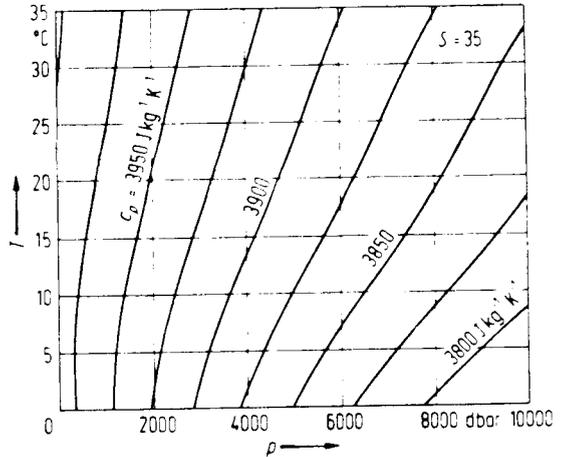


Fig. 13. Specific heat at constant pressure c_p in $[\text{J kg}^{-1} \text{ K}^{-1}]$ as function of pressure and temperature for $S = 35$. Contour interval $25 \text{ J kg}^{-1} \text{ K}^{-1}$.

The specific heat at constant volume c_v can be obtained from the thermodynamic relation

$$c_v = c_p - \frac{T_s v \alpha^2}{\chi_T} \tag{48}$$

For χ_T in $[\text{dbar}^{-1}]$ and SI-units otherwise (i.e. T_s in $[\text{K}]$, v in $[\text{m}^3 \text{ kg}^{-1}]$, α in $[\text{K}^{-1}]$, and c_p, c_v in $[\text{J kg}^{-1} \text{ K}^{-1}]$:

$$c_v = c_p - 10^4 \frac{T_s v \alpha^2}{\chi_T} \tag{49}$$

The corresponding formula results from introducing the specific volume $v(S, T, p)$ corresponding to eqs. (13)–(22), the thermal expansion coefficient $\alpha(S, T, p)$, eqs. (38)–(43), and the isothermal compressibility $\chi_T(S, T, p)$, eq. (33).

Check values:

$$\begin{aligned} S = 40, \quad T = 35^\circ \text{C}, \quad p = 0 \text{ dbar} \\ c_v = 3876.9516 \text{ J kg}^{-1} \text{ K}^{-1}, \\ S = 40, \quad T = 35^\circ \text{C}, \quad p = 10000 \text{ dbar} \\ c_v = 3690.2412 \text{ J kg}^{-1} \text{ K}^{-1}. \end{aligned}$$

Estimate of accuracy:

With the difference of c_p and c_v being small, c_v has the same uncertainty as c_p .

The ratio of specific heats $\gamma = c_p/c_v$ is given by

$$\gamma = \left[1 - \frac{T_s v \alpha^2}{\chi_T c_p} \right]^{-1} \tag{50}$$

and can be combined from the respective eqs. (13)–(22), (38)–(43), and (33). For the units used in this section a factor of 10^4 occurs with the last term in eq. (50).

Check value:

$$\begin{aligned} S = 40, \quad T = 35^\circ \text{C}, \quad p = 10000 \text{ dbar} \\ \gamma = 1.0341112. \end{aligned}$$

Estimate of accuracy:

The accuracy may be roughly estimated as $\Delta\gamma = 10^{-4}$ using $\Delta\alpha/\alpha = 5 \cdot 10^{-3}$, the largest error occurring in eq. (50).

The ratio γ for atmospheric pressure is represented in Fig. 14, and for $S=35$ at varying temperature and pressure in Fig. 15.

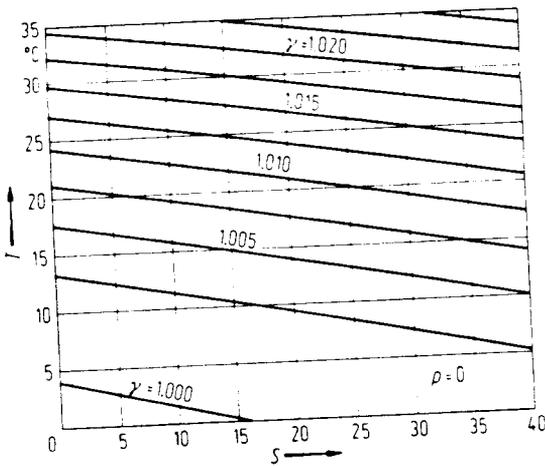


Fig. 14. Ratio of specific heats at constant pressure and constant volume $\gamma = c_p/c_v$ as function of salinity and temperature at atmospheric pressure. Contour interval $2.5 \cdot 10^{-3}$.

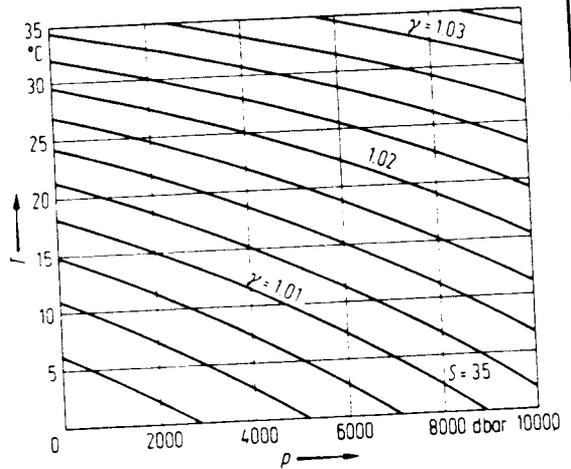


Fig. 15. Ratio of specific heats at constant pressure and constant volume $\gamma = c_p/c_v$ as function of pressure and temperature at $S=35$. Contour interval $2.5 \cdot 10^{-3}$.

3.1.7 Adiabatic lapse rate, potential temperature

The adiabatic lapse rate Γ is the change of temperature due to a unit adiabatic pressure change. It can be obtained from direct measurements or from:

$$\Gamma = \frac{T_a}{c_p} \left(\frac{\partial v}{\partial T} \right)_{S,p} \quad (51)$$

The formula of Bryden [73 B] resulting from the above equation is given here because it is consistent with EOS-80 (see subject. 3.1.3).

$$\Gamma(S, T, p) = \sum_{i=0}^3 a_i T^i + (b_0 + b_1 T)(S - 35) + p \left[\sum_{i=0}^3 c_i T^i + (d_0 + d_1 T)(S - 35) \right] + p^2 \sum_{i=0}^2 e_i T^i \quad (52)$$

- | | | |
|--------------------------------|--------------------------------|--------------------------------|
| $a_0 = 3.5803 \cdot 10^{-5}$ | $c_0 = 1.8741 \cdot 10^{-8}$ | $e_0 = -4.6206 \cdot 10^{-13}$ |
| $a_1 = 8.52587 \cdot 10^{-6}$ | $c_1 = -6.7795 \cdot 10^{-10}$ | $e_1 = 1.8676 \cdot 10^{-14}$ |
| $a_2 = -6.83605 \cdot 10^{-8}$ | $c_2 = 8.7330 \cdot 10^{-12}$ | $e_2 = -2.1687 \cdot 10^{-16}$ |
| $a_3 = 6.6228 \cdot 10^{-10}$ | $c_3 = -5.4481 \cdot 10^{-14}$ | |
| $b_0 = 1.8932 \cdot 10^{-6}$ | $d_0 = -1.1351 \cdot 10^{-10}$ | |
| $b_1 = -4.23935 \cdot 10^{-8}$ | $d_1 = 2.7759 \cdot 10^{-12}$ | |

Units:

S	T	p	Γ
ISS-69			
	C	dbar	K dbar ⁻¹

Range of validity:

$$\begin{aligned} 30 &\leq S \leq 40, \\ 2^\circ\text{C} &\leq T \leq 30^\circ\text{C}, \\ 0 &\leq p \leq 10000 \text{ dbar}. \end{aligned}$$

Check value:

$$\begin{aligned} S &= 40, \quad T = 30^\circ\text{C}, \quad p = 10000 \text{ dbar} \\ \Gamma &= 2.8774579 \cdot 10^{-4} \text{ K dbar}^{-1}. \end{aligned}$$

Estimate of accuracy:

Systematic errors range from $\pm 2.7 \cdot 10^{-6} \text{ K dbar}^{-1}$ to $\pm 5.3 \cdot 10^{-6} \text{ K dbar}^{-1}$.

Caldwell and Eide [80 C 1] have given an algorithm for Γ based on direct measurements which covers a larger range of validity.

The potential temperature Θ is the temperature resulting from an adiabatic displacement of an element of sea water at pressure p and temperature T to a reference pressure p_r . It can be obtained from

$$\Theta(S, T, p, p_r) = T + \int_p^{p_r} \Gamma(S, T', p') dp' \quad (53)$$

with integration along an adiabat, using Γ as given above. T' and p' are temperature and pressure, respectively, along the adiabatic line. It is common to select $p_r = 0$. Following Fofonoff [77 F], a Runge-Kutta integration algorithm is given which is superior to the available polynomial relation (Bryden [73 B]).

Potential temperature is given by:

$$\begin{aligned} \Delta\theta_1 &= \Delta p \Gamma(S, T, p) & \theta_1 &= T + \Delta\theta_1/2 \\ \Delta\theta_2 &= \Delta p \Gamma(S, \theta_1, p + \Delta p/2) & \theta_2 &= \theta_1 + (1 - \sqrt{2}/2)(\Delta\theta_2 - q_1) \\ \Delta\theta_3 &= \Delta p \Gamma(S, \theta_2, p + \Delta p/2) & \theta_3 &= \theta_2 + (1 + \sqrt{2}/2)(\Delta\theta_3 - q_2) \\ \Delta\theta_4 &= \Delta p \Gamma(S, \theta_3, p + \Delta p/2) & \theta_4 &= \theta_3 + (\Delta\theta_4 - 2q_3)/6 \\ q_1 &= \Delta\theta_1 \\ q_2 &= (2 - \sqrt{2})\Delta\theta_2 + \left(-2 + \frac{3}{\sqrt{2}}\right)q_1 \\ q_3 &= (2 + \sqrt{2})\Delta\theta_3 + \left(-2 - \frac{3}{\sqrt{2}} - q_2\right) \\ \Delta p &= p_r - p. \end{aligned} \quad (54)$$

Then the potential temperature Θ is obtained from:

$$\Theta(S, T, p, p_r) = \theta_4.$$

Units.

Use consistent units, e.g. p in [dbar], Γ in [K dbar^{-1}].

Range of validity:

Δp may exceed 10000 dbar.

Check value [using Γ from eq. (52)]:

$$\begin{aligned} S &= 40, \quad T = 40^\circ\text{C}, \quad p = 10000 \text{ dbar}, \quad p_r = 0 \text{ dbar} \\ \Theta &= 36.89072^\circ\text{C}. \end{aligned}$$

Estimate of accuracy:

Integration error is less than 10^{-4} K for $\Delta p = 10000 \text{ dbar}$. Systematic error in conjunction with eq. (53) is less than 0.024 K for $\Delta p = 10000 \text{ dbar}$.

The potential temperature difference $\Delta\theta = \theta - T$ for $S = 35$ and $p_r = 0$ dbar is shown in Fig. 16.

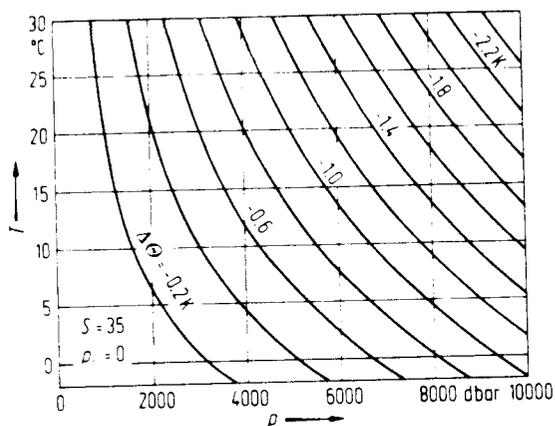


Fig. 16. Difference between potential and in-situ temperature in [K] as function of pressure and in-situ temperature; $S = 35$, $p_r = 0$. Contour interval 0.2 K.

3.1.8 Vapour pressure, boiling point, freezing point

Increasing salinity causes a lowering of vapour pressure at the water surface. This leads to a decrease in freezing temperature and an increase in boiling temperature.

An equation for the vapour pressure p_v of fresh water was derived from several investigators' data, taking account of IPTS-68, by Ambrose and Lawrenson [72 A]:

$$\log p_v = \left[\frac{a_0}{2} + \sum_{i=1}^{11} a_i E_i(x) \right] T_a^{-1}, \quad (55)$$

$$x = [2T_a - (T_{a2} + T_{a1})] / (T_{a2} - T_{a1}), \quad (56)$$

where p_v is the vapour pressure, T_a the thermodynamic temperature, and E_i the Chebyshev polynomial of degree i

$$\begin{aligned} a_0 &= 2794.0144 & a_3 &= 7.6875 & a_6 &= 0.1371 & a_9 &= 0.0200 \\ a_1 &= 1430.6181 & a_4 &= -0.0328 & a_7 &= 0.0629 & a_{10} &= 0.0117 \\ a_2 &= -18.2465 & a_5 &= 0.2728 & a_8 &= 0.0261 & a_{11} &= 0.0067 \\ T_{a2} &= 648 \text{ K}, & T_{a1} &= 273 \text{ K}. \end{aligned}$$

Units:

T_a IPTS-68	p_v
K	kPa or 10^{-1} dbar

Range of validity:

$$273.15 \text{ K} \leq T_a \leq 647.31 \text{ K}.$$

Check value:

$$\begin{aligned} T_a &= 647.31 \text{ K} \\ p_v &= 2.2106469 \cdot 10^4 \text{ kPa}. \end{aligned}$$

Estimate of accuracy:

Not available in original literature.

The change Δp_v in vapour pressure due to sea salt is described by Millero [83 M]:

$$\Delta p_v = AS + BS^{3/2} + CS^2, \quad (57)$$

$$A = -3.7433 \cdot 10^{-3} - 1.6537 \cdot 10^{-4} T - 1.9667 \cdot 10^{-6} T^2 - 2.4350 \cdot 10^{-7} T^3, \quad (58)$$

$$B = 5.2556 \cdot 10^{-4} - 7.7266 \cdot 10^{-6} T^3, \quad (59)$$

$$C = -4.9535 \cdot 10^{-5}. \quad (60)$$

Units:

S	T	Δp_v
ISS-69	IPTS-68	
°C		Pa or 10^{-4} dbar

Range of validity:

$$0 \leq T \leq 40 \text{ C}$$

$$0 \leq S \leq 40.$$

Estimate of accuracy:

Standard deviation of fit: 0.13 Pa; values of Δp_v are derived indirectly, thus a number of the accuracy cannot easily be given.

Check value:

$$S = 40, \quad T = 40 \text{ °C}$$

$$\Delta p_v = -1.5836562 \cdot 10^2 \text{ Pa}.$$

The graph in Fig. 17 represents the change in vapour pressure as a function of salinity and temperature.

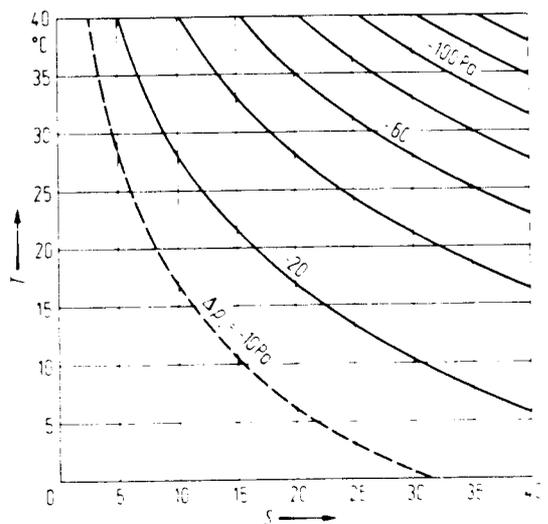


Fig. 17. Lowering of vapour pressure due to sea salt in [Pa] as function of salinity and temperature. Full lines: Contour interval 20 Pa. Dashed line: $\Delta p_v = -10$ Pa.

The boiling point temperature T_b is obtained by inverting

$$p_v = p_v(S, T) \quad \text{to} \quad T_b = T_b(S, p_v)$$

which can be done numerically. For high temperatures and high salinities the boiling point elevation ΔT_b due to sea salt was given by Stoughton and Lietzke [67 S]

$$\Delta T_b = I(A + BI + CI^2)D^{-1}, \quad (61)$$

where

$$\begin{aligned} A &= 565.757 T_a^{-1} - 9.81559 + 1.54739 \ln T_a \\ B &= -337.178 T_a^{-1} + 6.41981 - 0.922753 \ln T_a \\ C &= 32.681 T_a^{-1} - 0.55368 + 0.079022 \ln T_a \\ D &= 266919.6 T_a^{-2} - 379.669 T_a^{-1} + 0.334169 \end{aligned}$$

and where the ionic strength

$$I = \frac{1.9915 \cdot 10^{-2} S}{1 - 1.00487 \cdot 10^{-3} S} \quad (62)$$

Units:

S	T_a IPTS-68	ΔT_b
	K	K

Range of validity:

$$\begin{aligned} 20 &\leq S \leq 280, \\ 298 \text{ K} &\leq T_a \leq 530 \text{ K}. \end{aligned}$$

Check value:

$$S = 280, \quad T_a = 530 \text{ K}: \quad \Delta T_b = 15.61977 \text{ K}.$$

Estimate of accuracy:

$\pm 2\%$ relative error at low temperature and low concentration, $\pm 1\%$ at $300 \text{ K} \leq T_a \leq 520 \text{ K}$ and $34.5 \leq S \leq 200$.

In determining the ionic strength, the equation given by Millero and Leung [76 M 3] for the relation between the portion of dissolved solids and salinity was introduced here.

The boiling point temperature as a function of salinity and pressure is shown in Fig. 18.

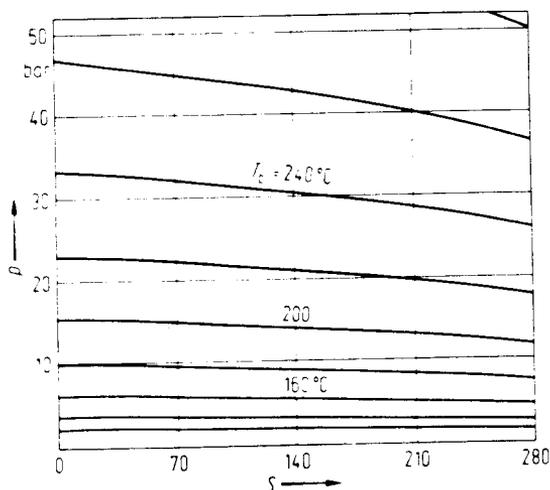


Fig. 18. Boiling point temperature T_b of concentrated sea salt solutions in $^{\circ}\text{C}$ as function of salinity and pressure. Contour interval 20°C .

From several independent measurements of the freezing point dependence on salinity and pressure, Millero [78 M] selected the data of Doherty and Kester [74 D] to establish the following equation for the freezing point temperature T_f . In this case salinities correspond to ISS-69 and not to PSS-78, and the pressure dependence is sufficiently accurate only for oceanic salinities.

$$T_f = -0.0575 S + 1.710523 \cdot 10^{-3} S^{3/2} - 2.154996 \cdot 10^{-4} S^2 - 7.53 \cdot 10^{-4} p. \quad (63)$$

Units:

S	p	T_f
ISS-69		IPTS-68
	dbar	°C

Range of validity:

$$0 \leq S \leq 40$$

$$0 \leq p \leq 500 \text{ dbar.}$$

Check value:

$$S = 40, \quad p = 500 \text{ dbar}$$

$$T_f = -2.588566 \text{ °C.}$$

Accuracy:

$$\pm 0.004 \text{ °C.}$$

Fig. 19 represents the freezing point temperature as a function of salinity at atmospheric pressure and as a function of pressure for $S = 35$. The cross-over point for the freezing point temperature and the density maximum temperature T_m (see subject. 3.1.5) at atmospheric pressure is found at $S = 23.6$ with an uncertainty of ± 0.2 . When T_m is derived from EOS-80 a value of $S = 24.6$ results.

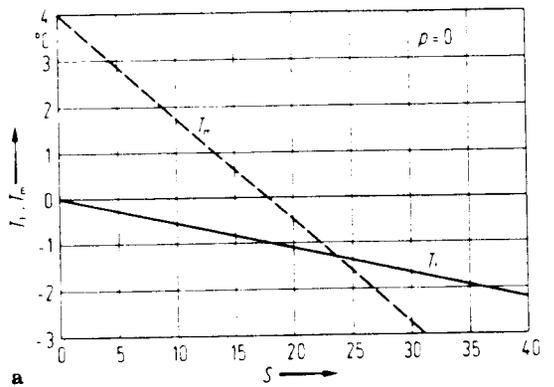


Fig. 19a. Freezing point temperature T_f as function of salinity at atmospheric pressure and maximum density temperature T_m .

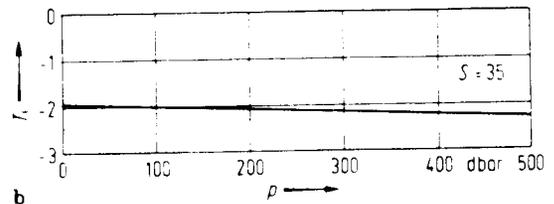


Fig. 19b. Freezing point temperature as function of pressure for $S = 35$.

3.1.9 Heat of fusion, heat of evaporation

The heat of fusion is the amount of heat required to transform a unit mass from the solid to the liquid state. The heat of fusion of fresh water at atmospheric pressure is $3.334 \cdot 10^5 \text{ J kg}^{-1}$ according to Landolt-Börnstein, 6th ed., vol. II, part 4, p. 180 [61 S]. It is not possible to give a simple function of salinity for sea water because fusion and crystallization of sea ice do not occur at a fixed temperature, but in a range of temperatures below the freezing point (Doronin and Kheisin [77 D]). The heat required for the complete fusion of sea ice is given in ch. 8.

The heat of evaporation or latent heat L is the amount of heat required to transform a unit mass from the liquid to the gaseous state. For fresh water it depends on temperature, and a least-squares fit to data from Osborne et al. [39 O] yields the following formula for the heat of evaporation for fresh water:

$$L(T) = 2.50039 \cdot 10^6 - 2.3683 \cdot 10^3 T + 4.31 \cdot 10^{-1} T^2 - 1.131 \cdot 10^{-2} T^3 \quad (64)$$

Units:

T	L
$^{\circ}\text{C}$	J kg^{-1}

Range of validity:

$$0 \leq T \leq 100^{\circ}\text{C}.$$

Check value:

$$T = 100^{\circ}\text{C}: \quad L = 2.2566 \cdot 10^6 \text{ J kg}^{-1}.$$

Estimate of accuracy:

Osborne et al. [39 O] do not give any estimate of accuracy. The standard deviation of eq. (64) from their data is 140 J kg^{-1} . According to [61 S] $L(100^{\circ}\text{C}) = 2.257 \cdot 10^6 \text{ J kg}^{-1}$. The above formula is not based on IPTS-68, however, deviations caused by the choice of temperature scales are well within accuracy limits.

No formulas are available for the change of the heat of evaporation as function of salinity. The deviation $\Delta L(S, T)$ from $L(T)$ for selected values can be derived from Stoughton and Lietzke [67 S]:

$$S = 34.33, \quad T = 25^{\circ}\text{C}: \quad \Delta L = 2.57 \cdot 10^3 \text{ J kg}^{-1}$$

$$S = 34.33, \quad T = 100^{\circ}\text{C}: \quad \Delta L = 3.18 \cdot 10^3 \text{ J kg}^{-1}.$$

Their paper contains data on the minimum energy necessary for the recovery of water from sea water.

3.1.10 Thermal conductivity

The thermal conductivity λ is a measure of heat flux q per unit cross section for a given temperature gradient:

$$q = -\lambda \text{ grad } T. \quad (65)$$

The following formulas for the thermal conductivity are recommended:

$$\lambda(S, T, p) = \lambda(S, T, 0) [1 + p f(S, T, p)]. \quad (66)$$

$$\lambda(S, T, 0) = 0.565 + (1.8 \cdot 10^{-3} - 3.2 \cdot 10^{-6} S) T - 9.5 \cdot 10^{-5} S - 5.9 \cdot 10^{-6} T^2, \quad (67)$$

$$f(S, T, p) = 6.9 \cdot 10^{-6} - 8 \cdot 10^{-9} T - 2 \cdot 10^{-15} p - 10^{-8} S. \quad (68)$$

Units:

S	T	p	f	λ
ISS-69	IPTS-68			
$^{\circ}\text{C}$		dbar	dbar^{-1}	$\text{W m}^{-1} \text{K}^{-1}$

Range of validity:

$$0 \leq S \leq 45, \quad 0 \leq T \leq 60^\circ\text{C},$$

$$0 \leq p \leq 14000 \text{ dbar}.$$

Check value:

$$S = 45, \quad T = 60^\circ\text{C}, \quad p = 14000 \text{ dbar}$$

$$\lambda = 0.6897343 \text{ W m}^{-1} \text{ K}^{-1}.$$

Estimate of accuracy:

Overall accuracy $\pm 1.5\%$. Internal consistency better than $\pm 0.005 \text{ W m}^{-1} \text{ K}^{-1}$. The accuracy of the coefficients in eqs. (67) and (68) is low.

Eqs. (66) and (68) are given by Caldwell [74 C 1], while eq. (67) reduces to Riedel's formula (Powell [58 P]) for fresh water. For $S=31.5$ it differs from Caldwell's formula [74 C 1] by a constant offset of $-0.004 \text{ W m}^{-1} \text{ K}^{-1}$. Within the accuracy stated and for the range of validity given, the above algorithm is consistent with Castelli et al. [74 C 3] and Jamieson and Tudhope [70 J]. The last paper gives a formula for $\lambda(S, T, 0)$ which covers wider ranges in S and T .

Fig. 20 displays λ as function of temperature for $p=0$ and $S=35$. The pressure and temperature dependence is represented in Fig. 21 for $S=35$.

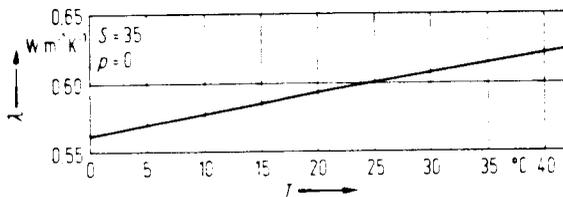


Fig. 20. Thermal conductivity λ in $[\text{W m}^{-1} \text{ K}^{-1}]$ for $S=35$ as function of temperature at atmospheric pressure.

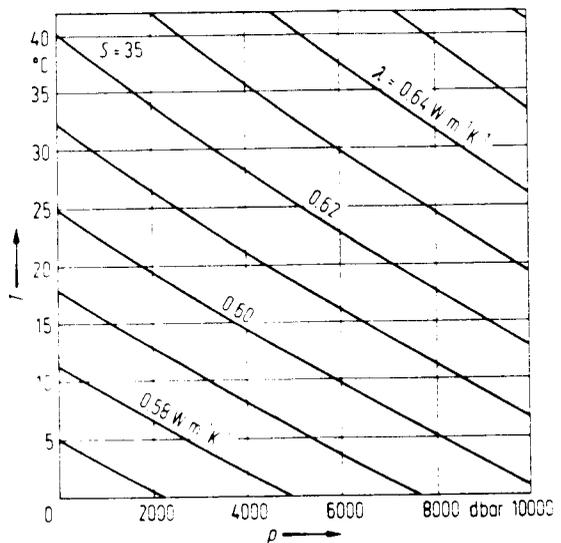


Fig. 21. Thermal conductivity λ in $[\text{W m}^{-1} \text{ K}^{-1}]$ as function of pressure and temperature at $S=35$. Contour interval $10^{-2} \text{ W m}^{-1} \text{ K}^{-1}$.

3.1.11 Diffusion

The diffusion coefficient is a measure of the flux of substance per unit cross section. The flux depends on the concentration gradient of the substance (Fickian diffusion), but also to a lesser extent on the temperature gradient (Soret effect, thermal diffusion). The flux J of the substance for a concentration n (mass of solute per unit mass of solution) is given by:

$$J = -\rho D \nabla n - \rho D_s n(1-n) \nabla T, \quad (69)$$

where D denotes the Fickian diffusion coefficient, and D_s the Soret coefficient.

It is not possible to determine D and D_s as functions of salinity since individual salts have different coefficients. Results are given here for the diffusion coefficient of a NaCl 0.5 normal aqueous solution (Caldwell [73 C, 74 C 2]) corresponding to a concentration $n = 28.5 \cdot 10^{-3}$, which would be found in sea water with $S=32.9$.

$$D = 10^{-11} (62.5 + 3.63 T). \quad (70)$$

Units:

$$\begin{array}{cc} T & D \\ \text{IPTS-68} & \\ \hline ^\circ\text{C} & \text{m}^2\text{s}^{-1} \end{array}$$

Range of validity:

$$2^\circ\text{C} \leq T \leq 52^\circ\text{C}.$$

The pressure dependence of D is small ($0 \leq p \leq 7000$ dbar).

Check value:

$$T = 52^\circ\text{C}; \quad D = 2.513 \cdot 10^{-9} \text{m}^2\text{s}^{-1}.$$

Estimate of accuracy:

The standard deviation of the measurements from the formula is $9 \cdot 10^{-11} \text{m}^2\text{s}^{-1}$.

Caldwell [74 C 2] has given the following formula for the Soret coefficient of a 0.5 normal solution of NaCl:

$$D_s = 10^{-5} [-119 + 11.05 T - 0.064 T^2 + 10^{-4} p(320 - 8.48 T + 0.0532 T^2)]. \quad (71)$$

Units:

$$\begin{array}{ccc} T & p & D_s \\ \text{IPTS-68} & & \\ \hline ^\circ\text{C} & \text{dbar} & \text{K}^{-1} \end{array}$$

Range of validity:

$$\begin{array}{l} 2^\circ\text{C} \leq T \leq 52^\circ\text{C} \\ 0 \leq p \leq 7000 \text{ dbar}. \end{array}$$

Check value:

$$T = 50^\circ\text{C}, \quad p = 7000 \text{ dbar}; \quad D_s = 2.938 \cdot 10^{-3} \text{K}^{-1}.$$

Estimate of accuracy:

The standard deviation of the measurements from the formula is $5 \cdot 10^{-5} \text{K}^{-1}$. Caldwell [75 C] showed that the pressure dependence of D_s for a 0.1 normal solution of NaCl at 17.65°C is significantly nonlinear.

3.1.12 Viscosity

In a fluid with molecular friction, the tangential stress τ is proportional to the shear U' . The proportionality factor is called (molecular) viscosity μ

$$\tau = \mu U'. \quad (72)$$

Often the factor divided by density is used, the kinematic viscosity $\nu = \mu/\rho$. For distilled water, $\mu(20^\circ\text{C}) = (1.002 \pm 0.0003) \cdot 10^{-3} \text{Ns m}^{-2}$ (Swindells et al. [52 S]).

According to Korson et al. [69 K] the viscosity of fresh water at atmospheric pressure can be calculated from

$$\log \frac{\mu(T)}{\mu(20^\circ\text{C})} = \frac{a(20-T) - b(T-20)^2}{T+c}, \quad (73)$$

where

$$a = 1.1709, \quad b = 1.827 \cdot 10^{-3}, \quad c = 89.93$$

and T is in [C].

Check value:

$$T = 70^\circ\text{C}: \quad \mu = 4.038703 \cdot 10^{-4} \text{ N s m}^{-2},$$

$$\text{with } \mu(20^\circ\text{C}) \text{ as given above, } \mu(20^\circ\text{C}) = 1.002 \cdot 10^{-3} \text{ N s m}^{-2}.$$

Range of validity:

$$10^\circ\text{C} \leq T \leq 70^\circ\text{C}.$$

Estimate of accuracy:

The maximum deviation from previously reported values is $5.1 \cdot 10^{-6} \text{ N s m}^{-2}$ at low temperatures.

According to Millero [74 M, p. 65f.] and Riley and Skirrow [75 R, p. 576] the relative viscosity of sea water at atmospheric pressure can be obtained from

$$\mu_s = \frac{\mu(S, T)}{\mu(0, T)} = 1 + A Cl_t^{1.2} + B Cl_t \quad (74)$$

$$A = 5.185 \cdot 10^{-5} T + 1.0675 \cdot 10^{-4}$$

$$B = 3.300 \cdot 10^{-5} T + 2.591 \cdot 10^{-3}.$$

Cl_t is the volume chlorinity, which is related to salinity by

$$Cl_t = \frac{\rho S}{1806.55} \quad (75)$$

while $\mu(0, T) = \mu(T)$, eq. (73).

Units:

S	T	ρ
ISS-69	IPTS-68	
	C	kg m^{-3}

Range of validity:

$$0 \leq S \leq 40$$

$$5^\circ\text{C} \leq T \leq 25^\circ\text{C}.$$

Check value:

$$S = 40, \quad T = 25^\circ\text{C}$$

$$\mu = 9.654093 \cdot 10^{-4} \text{ N s m}^{-2},$$

$$\text{with } \mu(0, T) \text{ as given above, eq. (73).}$$

Estimate of accuracy:

Not available in original literature.

For low temperatures an algorithm for the pressure dependence of viscosity is available only for standard sea water ($S = 35$). According to Matthäus [72 M]:

$$\mu(35, T, p) = \mu(35, T, 0) + \Delta\mu, \quad (76)$$

$$\Delta\mu = p_s [a_1 + a_2 T + a_3 T^2 + p_s (a_4 + a_5 T + a_6 T^2)], \quad (77)$$

where

$$p_s = p + 10.13$$

$$a_1 = -1.8266 \cdot 10^{-8}$$

$$a_2 = 1.3817 \cdot 10^{-9}$$

$$a_3 = -2.6362 \cdot 10^{-11}$$

$$a_4 = 9.8965 \cdot 10^{-13}$$

$$a_5 = -6.325 \cdot 10^{-14}$$

$$a_6 = 1.2115 \cdot 10^{-15}.$$

Units:

T IPTS-68	p	$\Delta\mu$
$^{\circ}\text{C}$	dbar	N s m^{-2}

Range of validity:

$$0 \leq T \leq 20^{\circ}\text{C},$$

$$0 \leq p \leq 10000 \text{ dbar}$$

$$20^{\circ}\text{C} \leq T \leq 30^{\circ}\text{C} \text{ for low pressure only.}$$

The algorithm eqs. (73)–(77) can be used for salinities differing from 35 without introducing much error. At 10°C and 10000 dbar a value is obtained for fresh water which is about $4 \cdot 10^{-5} \text{ N s m}^{-2}$ too high.

Check value:

$$S = 35, \quad T = 10^{\circ}\text{C}, \quad p = 10000 \text{ dbar}$$

$$\mu = 1.363401 \cdot 10^{-3} \text{ N s m}^{-2},$$

with $\mu(35, T, 0)$ as given above, eqs. (73)–(75).

Estimate of accuracy:

The formula deviates from the measurements by less than $\pm 4 \cdot 10^{-6} \text{ N s m}^{-2}$. Stanley and Batten [69 S] estimate the accuracy of their data as $\pm 1.5 \cdot 10^{-5} \text{ N s m}^{-2}$.

The viscosity for atmospheric pressure is shown in Fig. 22, and Fig. 23 represents the viscosity for $S = 35$ and varying temperature and pressure.

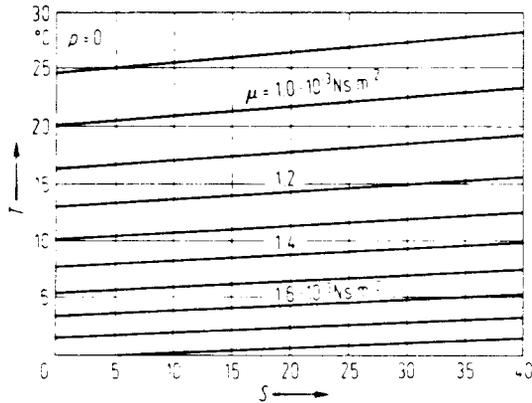


Fig. 22. Viscosity μ in $[10^{-3} \text{ N s m}^{-2}]$ as function of salinity and temperature at atmospheric pressure. Contour interval $10^{-4} \text{ N s m}^{-2}$.

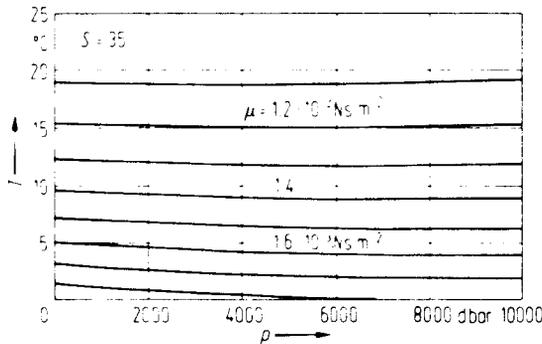


Fig. 23. Viscosity μ in $[10^{-3} \text{ N s m}^{-2}]$ as function of pressure and temperature at $S = 35$. Contour interval $10^{-4} \text{ N s m}^{-2}$.

3.1.13 Surface tension

If a free surface element is extended by δF , a work δA has to be performed to balance the surface tension:

$$\delta A = a\delta F. \quad (78)$$

Here a is the constant of surface tension. Data given in Landolt-Börnstein, 6th ed., vol. II, part 3, p. 421 [56 S], for distilled water can be represented by

$$a(T) = 7.562 \cdot 10^{-2} - 1.3928 \cdot 10^{-4} T - 3.063 \cdot 10^{-7} T^2. \quad (79)$$

Units:

T	a
IPTS-48	
$^{\circ}\text{C}$	N m^{-1}

Range of validity:

$$0 \leq T \leq 60^{\circ}\text{C}.$$

Check value:

$$T = 60^{\circ}\text{C}: \quad a = 6.616052 \cdot 10^{-2} \text{ N m}^{-1}.$$

Estimate of accuracy:

Not available in literature cited.

No recent measurements are available for sea water. According to Krümmel [01 K] a is about 1% lower for $S = 40$ than for fresh water. Due to surface films, mostly of organic origin, on the sea surface, large variations occur in the actually observed values of a .

3.1.14 Osmotic pressure

If water volumes with different salinities are separated by a porous wall that is permeable for water and not for salt, diffusion of water molecules will change salinities and cause a pressure difference. The pressure difference is the osmotic pressure Π . Following Millero and Leung [76 M 3] it can be calculated for sea water with salinity S relative to fresh water by the following equation:

$$\Pi = AS + BS^{3/2} + CS^2, \quad (80)$$

$$A = 0.070249 + 2.3938 \cdot 10^{-4} T - 3.7170 \cdot 10^{-7} T^2, \quad (81)$$

$$B = -2.1601 \cdot 10^{-3} + 4.8460 \cdot 10^{-7} T - 1.0492 \cdot 10^{-7} T^2, \quad (82)$$

$$C = 2.7984 \cdot 10^{-4} + 1.552 \cdot 10^{-6} T - 2.7048 \cdot 10^{-9} T^2. \quad (83)$$

Units:

S	T	Π
ISS-69	IPTS-68	
	$^{\circ}\text{C}$	MPa

Range of validity:

$$0 \leq S \leq 40$$

$$0 \leq T \leq 40^{\circ}\text{C}.$$

Check value:

$$S = 40, \quad T = 40^{\circ}\text{C}$$

$$\Pi = 3.1252952 \text{ MPa}.$$

Estimate of accuracy:

The standard deviation of the formula from the original data is $3 \cdot 10^{-4}$ MPa. Considerable deviations exist from earlier reported values.

For $p=0$ the osmotic pressure is represented in Fig. 24.

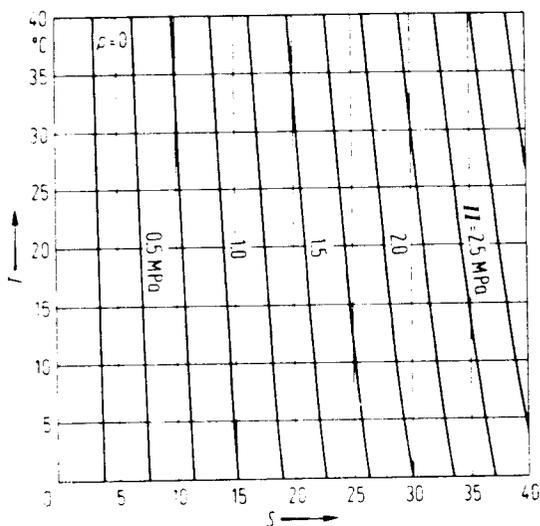


Fig. 24. Osmotic pressure Π in [MPa] as function of salinity and temperature at atmospheric pressure. Contour interval 0.25 MPa.

3.1.15 Brunt-Väisälä frequency

The Brunt-Väisälä frequency N (see sect. 6.1) is often calculated from the expression

$$N^2(p) = g^2 \left[\frac{d\rho}{dp} - c^{-2} \right], \quad (84)$$

where g denotes the gravitational acceleration, ρ the in-situ density. The term containing the sound velocity c accounts for the adiabatic effect. The algorithms used to compute $\rho(S, T, p)$ and $c(S, T, p)$ have to be thermodynamically consistent. This is largely true for Chen and Millero's formula for sound speed [77 C] and EOS-80 [eqs. (13)–(22)]. However, systematic errors due to the independent fitting procedures for ρ and c , and errors due to limited numerical precision lead to large relative errors in N for the deep ocean, where

$$\frac{d\rho}{dp} \approx c^{-2}.$$

Owens and Fofonoff (personal communication) proposed a method called adiabatic levelling. Finite differences are used instead of differentiation.

$$N^2(\bar{p}) = \bar{\rho}^2 g^2 \left[\frac{\delta(S_2, \theta(S_2, T_2, p_2, \bar{p}), \bar{p}) - \delta(S_1, \theta(S_1, T_1, p_1), \bar{p})}{p_2 - p_1} \right], \quad (85)$$

where δ is the specific volume anomaly [eqs. (26)–(28)], θ the potential temperature at reference pressure $\bar{p} = (p_2 + p_1)/2$, (e.g. eq. (54)), and $\bar{\rho}$ is to be calculated at pressure level \bar{p} .

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3.1.16 References for 3.1

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